# Design Methodologies

3

This chapter presents four primary design methodologies developed in the past several decades for increased design productivity and the resultant product quality. Although these methodologies are suitable and have been commonly targeted for the post-conceptual-design phase, some can also be of significant benefit during the conceptual design phase of a product. Axiomatic design methodology, for example, falls into this category.

Designers should attempt to use as many established design methodologies as possible during product development: For example, Axiomatic design and group technology at the conceptual design phase, design for manufacturing/assembly/environment guidelines during configuration and detailed design, and the Taguchi Method during parametric design.

# 3.1 AXIOMATIC DESIGN METHODOLOGY

As discussed in Chap. 2, the conceptual design phase starts with examining and identifying the customer's needs, which subsequently must be related to engineering requirements. Axiomatic design methodology, developed in the late 1970s by N. P. Suh, but only widely implemented since the 1980s, is primarily an analysis technique for the evaluation of designs. Users of this methodology can utilize its two axioms and their numerous corollaries as guidelines for good design.

The two axioms of the theory advocate that for good design the functional requirements (FRs) of the product (as dictated by the customer) must be independently satisfied by the design parameters (DPs), onto which they would be mapped, in the simplest possible manner. Suh defines the FRs as "a minimum set of independent requirements that completely characterize the functional needs of the product in the functional domain:"

Axiom 1: The independence axiom states that a change in a DP should preferably only affect its corresponding FR.

Axiom 2: The information axiom states that among all alternatives considered, which satisfy Axiom 1, the simplest solution is the best design.

As a way to categorize designs, Suh introduced the following design categories:

Uncoupled design: A concept that satisfies Axiom 1.

*Coupled design:* A concept that violates Axiom 1, where a perturbation in a DP affects multiple FRs.

*Decoupled design:* A concept that is initially a coupled design due to lack of sufficient DPs, but one that can be decoupled with the use of extra DPs.

A decoupled design would naturally have an information content that is more than that of an uncoupled (competing) design.

As a simple example, let us consider a user need for a control device for hot-water supply—that is, the device must control the flow rate as well as the temperature of the water. These can be defined as  $FR_1$  and  $FR_2$ , respectively. A possible design would be to have two knobs (DP<sub>1</sub>, DP<sub>2</sub>) individually controlling the flow rate of hot and cold water, respectively, prior to their mixing (Fig. 1a). As one can note, however, a user would have difficulty in achieving a desired output of a specific flow-rate set at a certain temperature using this design, necessitating numerous control interventions. This design concept can thus be classified as a coupled design. An uncoupled design would require that the individual DPs satisfy their corresponding FRs independently. Many such uncoupled design—based commercial products exist today, in which users can turn a lever (or a knob) left or right for a desired water temperature and tilt the lever up or down (or, pull or depress the knob) for flow-rate control (Fig. 1b).



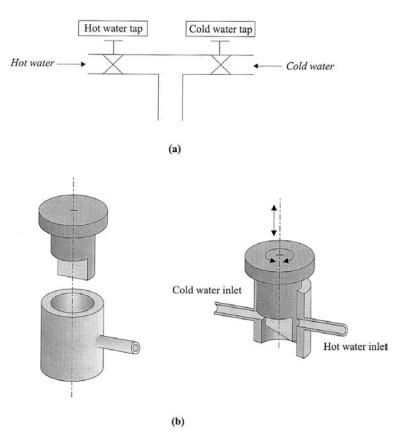


FIGURE 1 Axiomatic designs: (a) coupled and (b) uncoupled.

The mapping process between the FRs and the DPs can be expressed in a (linear) matrix form as

$$\{FR\} = [A]\{DP\} \tag{3.1}$$

where  $\{FR\}$  is the functional requirement vector and  $\{DP\}$  is the design parameter vector. The matrix [A] maps FRs into DPs.

An uncoupled design would have all the non diagonal elements of its [A] matrix as zero, thus satisfying the independence axiom. A coupled design, on the other hand, would have an [A] matrix with some nonzero nondiagonal elements. That is, some of the FRs will be functions of more than one DP.

For the first (coupled) water flow control device discussed above (Fig. 1a), the design matrix is

$$FR_{1} = A_{11}DP_{1} + A_{12}DP_{2}$$
  

$$FR_{2} = A_{21}DP_{1} + A_{22}DP_{2}$$
(3.2)

Thus, change in the two design parameters (both knobs) affects simultaneously both the flow rate  $(FR_1)$  and the temperature  $(FR_2)$ —the latter by proportioning the amount of water coming from two sources (hot and cold).

A coupled design can be decoupled by redesign: the [A] matrix becomes triangular (all elements above or below the diagonal have zero values):

$$\begin{cases} FR_1 \\ FR_2 \end{cases} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{cases} DP_1 \\ DP_2 \end{cases}$$
(3.3)

In the above equation, it is noted that  $FR_1$  is only a function of  $DP_1$ . Once the value of  $DP_1$  is set to correspond to a desired value of  $FR_1$ , subsequently,  $DP_2$  can be appropriately adjusted so that the combination of  $DP_1$  and  $DP_2$  yields a desired (functional requirement) value for  $FR_2$ , in which

$$FR_2 = A_{21}DP_1 + A_{22}DP_2 \tag{3.4}$$

When the number of FRs, m, is different from the number of DPs, n, the design is either coupled, m > n, or it is redundant in nature, m < n. A coupled design can be decoupled, first by the use of additional DPs, so that m becomes equal to n, and, subsequently by varying them (if necessary) so that the (square) mapping matrix, [A], becomes diagonal, normally, through a trial and error process. A redundant design, on the other hand, may not be a coupled design necessarily.

Let us consider the following case of m = 2 and n = 3:

$$\begin{cases} FR_1 \\ FR_2 \end{cases} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix} \begin{cases} DP_1 \\ DP_2 \\ DP_3 \end{cases}$$
(3.5)

In Eq. (3.5), if  $A_{13}$ ,  $A_{21}$ , and  $A_{22}$  are zero, FR<sub>1</sub> would be only a function of DP<sub>1</sub> and DP<sub>2</sub>, whereas FR<sub>2</sub> would only be a function of DP<sub>3</sub>, yielding functional independence. A preferable scenario might be to combine DP<sub>1</sub> and DP<sub>2</sub> into one design parameter to yield a more effective uncoupled design.

As one would expect, at the conceptual design phase, the objective of the designer is to note whether the elements of the mapping matrix [A] are zero or not. Once the design parameters (DPs) have been established, these will act as detailed design requirements and be mapped into specific variables for parametric design.

The minimization of information in a specific design, in order to satisfy Axiom 2, refers to its simplification process, for example, to increase its manufacturability. Design information may include geometric tolerances that are set realistically and material constraints/preferences and metallurgical treatments that should be chosen according to process availability and economic viability.

Suh provides a comprehensive list of design rules based on his two axioms, three of which are

Minimize the number of functional requirements and constraints. Use standardized components. Specify achievable tolerances.

The above should be seen as guidelines to be used in conjunction with numerous design rules to be specified in this chapter to satisfy objectives, such as manufacturability, ease of assembly, and environment friendliness.

### 3.2 DESIGN FOR X

Today it is commonly accepted that consideration of manufacturing and assembly issues during the design phase of a product is a fundamental part of concurrent engineering (CE). This was not the case in the first half of the 20th century, when CE was not a central manufacturing management policy, and designers were expected to be familiar with all manufacturing processes (and they actually were). In the latter half of the century (1950s to 1970s), though, manufacturing engineering was neglected as an undergraduate studies subject in the curricula of many North American universities; consequently most junior engineers lacked comprehensive knowledge of fabrication and assembly processes. Furthermore, these engineers became specialists in their fields (in the spirit of the Taylor/Ford paradigm) with little knowledge or appreciation of other disciplines. Thus the 1980s saw the necessary birth of CE-based design and the reintroduction of breadth into engineering curricula, so that engineers could communicate more effectively within their product design teams.

In this chapter, a limited number of design guidelines is presented for several manufacturing processes, for assembly, and for environmental considerations. The objective is to make the reader aware of the existence of such Design for X methodologies. The guidelines presented in the following subsections, though not comprehensive nor inclusive of all processes, are derivatives of the following general design guidelines:

- Design parts for ease of (and profitable) manufacturing—select materials and corresponding fabrication processes suitably.
- Specify tolerances, surface finish, and other dimensional constraints that are realistic.
- Minimize the number of parts, and furthermore use as many as standard parts as possible.
- Note that mechanical properties (and consequently a part's life) are affected by specific production process parameters, such as the location of parting lines in casting and molding.

# 3.2.1 Design for Manufacturing

Consideration of manufacturing (also termed as production or fabrication) processes during the design stage of a product is fundamental to successful design. Since selection of materials must precede consideration and analysis of manufacturability, herein it will be assumed that this stage is part of the definition of functional requirements (in response to customer needs) and thus will not be addressed. Primary issues that do arise during material selection include product life, environmental conditions, product features, and appearance factors.

In the following subsections, a select set of manufacturing processes will be reviewed, specifically from the perspective of design guidelines, in order to illustrate the importance of considering manufacturability during the product development stage. These and other design guidelines will be revisited when specific manufacturing processes are discussed in Part II of this book.

# Design for Casting

In casting, a molten metal is poured (or injected at high pressure) into a mold with a single or multiple cavities (Fig. 2). The liquid metal solidifies within the cavity and is normally subject to shrinkage problems. A good mold design will thus have features to compensate appropriately for shrinkage and avoid potential defects.

Castings must be designed so that parts (and patterns for sand casting) can be removed easily from the cavities. Different sections of a part with varying thicknesses should have gradual transitions. Projecting details should be avoided. Ribs should not be allowed to cross each other but should be offsett. In die casting, parts should have thin-walled structures to

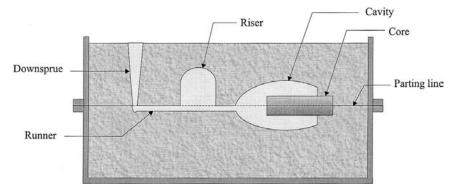


FIGURE 2 Sand casting.

ensure smooth metal flow and minimum distortion due to shrinkage. One should also note that die casting is normally limited to nonferrous metals. Furthermore, different casting techniques yield different dimensional accuracy and surface finish. For example, although sand casting can be used for any type of metal, poor surface finish and low dimensional accuracy are two of its disadvantages.

# Design for Forging

Forging is the most common (discrete-part) metal forming process in which normally a heated workpiece is formed in a die cavity under great (impact) pressure. Owing to the high forces involved, generally a workpiece is formed in multiple iterations (Fig. 3). As with casting, vertical surfaces of a part

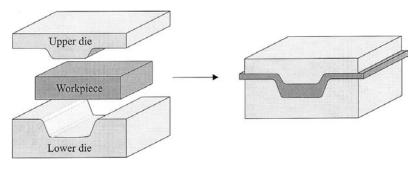


FIGURE 3 Closed-die forging.

must be tapered for ease of removal (normally done manually). Furthermore, rapid changes in section thicknesses should be avoided to prevent potential cracks. Finally, when designing a product that is to be forged, the location of the "parting line," where the two die halves meet, should be carefully chosen to influence positively the grain flow and thus the mechanical properties of the part.

#### Design for Machining

There is a variety of material-removal processes that are collectively called machining. Although both metals and plastics can be machined, for example, using turning, milling, and grinding operations, machining is primarily reserved for metal workpieces. Cylindrical (rotational) geometries can be obtained using a lathe or a boring machine (for internal turning), whereas prismatic (nonrotational) geometries can be obtained on a milling machine (Fig. 4). A drill press is reserved for making holes in prismatic objects. Although there are many other material-removal techniques, they will be discussed only in Part II of this book.

Machining is a flexible manufacturing operation in which metalcutting parameters can be carefully controlled to produce almost any external detail on a (one-of-a-kind) part, including 3-D complex surfaces. Automated machine tools can be programmed to fabricate parts in large quantities as well, such as nuts, bolts, and gears. Being material-removal techniques, such processes can take long periods of time when high accuracies are required and/or material hardnesses are very high. Thus a designer must carefully consider configuring features on a product that would require several setup activities, to rotate and realign the part, and subsequently prolong manufacturing times.

A common error in designing parts for machining is placement of holes (or other details) on a workpiece that would not be accessible due to collision between the tool-holder and the part (or even the fixture that holds the part) (Fig. 5).

Numerous design guidelines for machining have been described by Boothroyd et al., some of which are

- Preshape parts through casting or forging to minimize machining time. Avoid specifying features or tolerances that your machine tools cannot profitably fabricate.
- Ensure that the workpiece can be rigidly fixtured to withstand common high cutting forces.
- Avoid internal features in long workpieces (including cylindrical bores).

Avoid dimensional ratios that are very high.



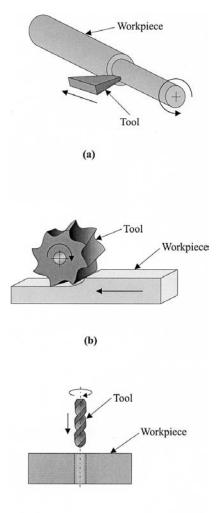




FIGURE 4 (a) Turning; (b) milling; (c) drilling.

# Design for Injection Molding

Injection molding is the most common plastic-parts manufacturing process for thin-walled objects. It commonly utilizes (recyclable) thermoplastic polymer granules that are melted and forced into a mold cavity (Fig. 6). It is a very efficient process in which multicavity molds (up to 16 or more) can manufacture several thousands of parts per hour and last for several

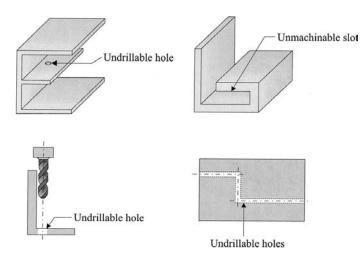


FIGURE 5 Unmachinable parts.

millions of parts. The three-step fabrication process comprises injection of molten plastic into the cavities, cooling (solidification) through the cavity walls, and forced ejection (for a typical total cycle time of 10–60 s).

As in casting and forging, the two most important design considerations in injection molding are wall thicknesses and parting lines. One always aims for gradual wall thickness changes through the part (typically, several mm) as well for incorporating as many features as possible into the design (snap fits, countersinks, holes, bosses, etc.) to avoid secondary operations. Other design guidelines proposed by Boothroyd et al. include

Configure your part geometry for adequate tapers for easy ejection from the cavities.

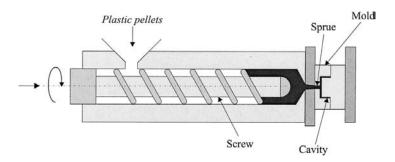


FIGURE 6 Injection molding.

- Ensure proper proportioning of wall thicknesses for minimum distortion during cooling.
- Minimize wall thicknesses (through the use of supporting elements) for fast cooling.
- Avoid depressions on the inner side surfaces of the part to simplify mold design and minimize cost.

#### 3.2.2 Design for Assembly

Assembly is a manufacturing process normally seen as an activity that does not add value to the final product. Thus every effort should be made to minimize assembly costs by minimization of the total number of parts, avoidance of several directions of assembly, and maximizing assemblability through the use of guidance features.

Most of the product design (for assembly) issues discussed below have been extensively reported in the pioneering works of G. Boothroyd, P. Dewhurst, and W. Knight since the 1970s. Although their work addressed the topics of manual and automatic assembly separately, such a distinction will not be made herein, since the emphasis of this book is on autonomous manufacturing systems; furthermore, most guidelines developed for the former case apply to the latter.

As providing a first level discussion to the three general design-forassembly guidelines provided above, this list addresses the issues of parts manipulation and joining (Fig. 7):

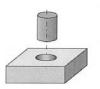
- Design parts with geometrical symmetry, and if not possible exaggerate the asymmetry.
- Avoid part features that will cause jamming and entanglement, and if needed add nonfunctional features to achieve this objective.
- Incorporate guidance features to part's geometry for ease of joining, such as chamfers; clearances should be configured for maximum guidance, but for minimum potential of jamming.
- Design products for unidirectional vertical (layered) assembly in order to avoid securing the previous subassembly while turning it.
- Incorporate joining elements into the parts (such as snap fits) in order to avoid holding them in place when utilizing additional joining elements (such as screws, bolts, nuts, or even rivets). Snap fits can be designed to allow future disassembly or be configured for permanent joining owing to potential safety hazards.

The major cost of assembly is determined by the number of parts in the product. Thus one should first and foremost attempt to eliminate as many parts as possible, primarily by combining them. Conditions for Do not

Do



Use snap-fits for easy joining







Use chamfers for easy insertion



Make parts symmetrical for easy orientation





Use layered product configuration for efficient assembly



Close ends to prevent entanglement

FIGURE 7 Assembly problems and solutions.

elimination can be recognized by examining the product: (1) Does the part move after the assembly, or simply remain static? (2) Why must the part be of a different material than the neighboring part? (3) Would the part prevent the assembly of other parts, by presenting an obstruction, if it were to be combined with a neighboring part?

As argued by the axiomatic design theory, eliminating parts from a product is in line with Axiom 2, which requires minimization of information (Sec. 1). Integration of parts (consolidation) may also reduce typical stress concentration points in parts owing to the use of external fasteners. (Naturally, the use of snap fits in product assembly also introduces stress concentration points, and thus their use should be carefully examined.) A beneficial side effect of part reduction is the elimination of future potential loosening in joints and subsequent vibration noise.

# 3.2.3 Design for the Environment

Human population growth is a major factor in the well-being of our environment, and when coupled with the complexity of our lifestyles it presents an enormous pressure on the world's precious resources. It is anticipated that in the 21st century, the world's population may peak at between 10 and 15 billion.

In the past century, the world's industrial production grew more than 100-fold. In the same period of time, the consumption of fossil fuel increased by a factor of more than 50. It has been eagerly argued that we cannot continue to use materials and resources at their current rates without experiencing severe shortages within the next 50 to 100 years.

No industrial activity today happens in isolation. It impacts the environment from the materials it uses to the products it manufactures, which have to be dealt with at the end of their life cycles. The approach to industrial-environmental interactions is commonly referred to as industrial ecology. It aims at designing industrial processes and products that minimize their impact on the environment, while maintaining manufacturing competitiveness. In that respect, one must be concerned about the following global issues: climate change, ozone depletion, loss of habitat and reductions in biodiversity, soil degradation, precipitation acidity, and degradation in water and air qualities.

A primary issue in industrial ecology is life cycle assessment (LCA), a formal approach to addressing the impact of a product on the environment as it is manufactured, used and finally disposed of. The first step of LCA is inventory analysis. That is, we need to determine the inputs (materials and energy) used in manufacturing and the outputs (the product itself, waste, and other pollutants) resulting during manufacturing and beyond. The second step of LCA is quantifying the impact of the outputs on the environment (a most contentious issue). The final step is the improvement analysis. Proposals are presented to manufacturers for reducing environmental impact—this stage is also referred as design for environment.

# Design for Energy Efficiency

The manufacturing industry uses a considerable amount of energy. For example, manufacturing activities consume almost 20% of electricity in the U.S.A. The answer to energy-source selection is not a simple one, because uses of different resources impact the environment at varied levels. As far as the atmosphere is concerned, for example, fossil-fuel combustion is more harmful than energy produced by nuclear power. That is, energy-source efficiency must often be balanced with toxicity concerns. However, no matter what its source of energy is, a manufacturing company must always aim for energy conservation when evaluating product design and fabrication process alternatives.

# Design for Minimum Residues

Numerous toxic chemicals are released to the environment during many of today's manufacturing processes. These residues can be solid, liquid, and/ or gaseous. For example, in the U.S.A., municipal solid waste discarded in landfills is less than 2% of the amount of industrial waste (of which more than half comes from manufacturing activities). Solid residues come in several forms: product residues generated during processing (for example, small pieces of plastic trimmings), process residues (such as cutting tools disposed of at the end of their useful life), and packaging residues (packaging and transportation material brought to the factory (drums, pallets, cardboard, etc.). A manufacturing company should make every effort to minimize all waste (recycle waste material as well as utilize reusable packaging).

# Design for Optimal Materials

Although it is a difficult issue to tackle, designers can make an effort in choosing product materials for minimal environmental impact, especially in relation to their extraction as well as processing. Naturally, an efficient recycling operation may provide manufacturers with adequate material supply with lower costs and minimal environmental impact. Thus recyclability of the product's material should be a factor in this - -

selection. Today metals are recycled with reasonable efficiency, returning them to their original condition through rework (remanufacture) or at worst by melting them. No matter what materials are utilized, one should always strive toward minimization of their amount through suitable engineering design (for example, by using thinner walls supported by many ribs).

#### Design for Recycling

In the past decade, besides the obvious economic benefits, manufacturers have also had to consider various government regulations (i.e., punitive incentives) when employing design-for-recycling practices. The common hierarchy of preferences in recycling practices available to manufacturing companies has been as follows (Fig. 8):

- *Subassemblies (highest)*: Replacement of a subassembly in a product in order to restore the product to its original operational level; the failed subassembly can also be recycled in order to restore it to its original performance level for future use.
- *Components:* Refurbishment of products first by their complete removal from operation, then by replacement of failed components with new or recycled components, and finally by their return to their normal operational level.
- *Materials (lowest):* Removal of products from operation, recycling of materials that can be recovered, and use of these materials (most frequently mixed with virgin materials) for the manufacturing of new components/products.

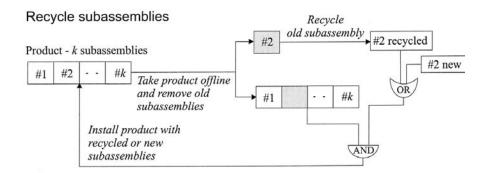
It is important to minimize the number of different materials used in the manufacturing of a product and, where possible, to keep them separated for ease of joining and subsequent recycling.

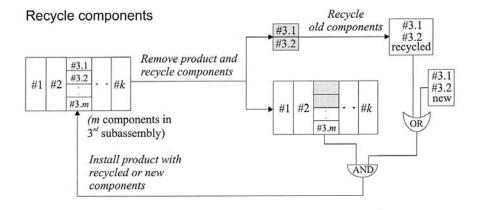
Another important issue is "design for disassembly." There exist two methods for common disassembly: reversible (where screws are removed, snap-fits unsnapped, etc.) and destructive (where the joints are broken). Economic and safety issues play major roles in deciding which joining technique to use. A modular design will greatly simplify the task of disassembly, as we can quickly identify the part/component/subassembly to be replaced (Chap. 2).

The design guidelines for "green" products and processes can be summarized as

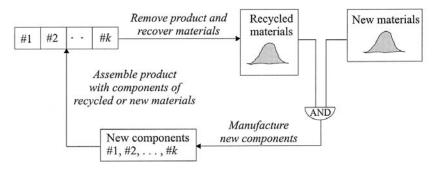
Increase efficiency of energy use, while considering environmental impact.

Minimize the amount of materials used.





# Recycle materials





Use recyclable and biodegradable materials where possible.

- Maximize the life expectancy of the product (in materials as well as technology).
- Design a modular product for ease of disassembly and remanufacturing,

#### 3.3 DESIGN OF EXPERIMENTS AND TAGUCHI'S METHOD

Parameter and tolerance design follows the conceptual design and engineering requirements determination phases of a product. At this stage, most engineers review functional requirements and decide on parameter and tolerance values based on experience, handbooks, etc. In the case of multiparameter design, however, where the choice of one parameter affects the other, engineers are advised to run experiments and optimize their values. Experimentation (for optimization) can be in the physical domain or in virtual space, where numerical simulations are performed.

#### 3.3.1 Parameter Design Using Design of Experiments and Response-Surface Optimization

It is strongly recommended that engineers take advantage of well-established statistical design of experiments (DOE) theories in order to minimize the search efforts for the optimal parameter values. The alternative would be to run a random (not well thought) set of experiments, from which one cannot easily infer meaningful conclusions. DOE theory advocates a factorial approach to experiments, that is, the controllable variables (parameters) of the experiment are discretized to a very limited number of levels (e.g., low, medium and high) and are randomized methodically in order to create a limited (but well thought) set of experiments. There exist a number of techniques for factorial design—the Latin square, the Youden square, and of course the Taguchi method.

As an example of a full-factorial design of experiments, let us assume that the fatigue failure level of a product depends on three (dimensional) parameters (A, B, and C). We design an experiment to evaluate these dependencies, and decide to test two levels (low and high) for every parameter, respectively. Table 1 illustrates the results of the experiments for a set of  $2^3 = 8$  experiments. (Naturally, if it is economically viable, one may decide to repeat the experiments several times, if they are physical in nature, in order to minimize the effect of noise on the observations.)

Once the experiments have been completed a design engineer's objective would be to search for the optimal values of the parameters (not only

A	В	С	Failure (cycles $ imes$ 10 <sup>6</sup> )
L	L	L	1.27
Н	L	L	1.29
L	Н	L	1.31
L	L	Н	1.28
Н	Н	L	1.58
Н	L	Н	1.29
L	Н	Н	1.41
Н	Н	Н	1.39

**TABLE 1** A Design-of-Experiments Example

among the specific levels tested during the particular set of a limited number of experiments run but also through the complete feasible search space). Prior to this stage one may examine the obtained results through an analysis-of-variance study, to determine whether some of the parameters have a low impact on the output. In the case of a large number of parameters, it would be wise to select quickly the appropriate values for these parameters and exclude them from future searches for the optimal values of the remaining parameters. Based on such an investigation, one would note that the variation of Parameter A in our example (Table 1) has a low impact on the value of the failure cycle and thus could be eliminated after choosing a suitable value for it.

Response surface (RS) methodology is a common technique that can be utilized to facilitate the search for the optimal parameter values. As the name implies, the first step of the RS methodology is to establish a (continuous-variable) relationship between the variables and the output (observation) through a surface fit (a hypersurface, if the number of variables is above two). Least-squares-based regression methods are commonly used for this purpose—namely, in fitting a response surface to experimentally obtained data. (Naturally, it is strongly advised that experimental data be collected using a DOE theory.) One must recall that, during the surface-fitting process, the actual numerical values of the variables that correspond to the two levels examined (e.g., L and H) are utilized. For our example above, Table 1, after eliminating Parameter A from the optimization, the outcome of the regression analysis would be a three-dimensional surface, if we assume a nonlinear relationship between the variables (B and C) and the output (failure cycles). - -

As the last step, a nonlinear constrained search method must be utilized in order to search effectively for the optimal values of the variables within the search space defined by the response surface.

Therefore in conclusion to the above discussion, we can summarize the three-step parameter-design phase as follows:

- 1. Use DOE theory for selecting a limited set of experiments (not necessarily full factorial).
- 2. Determine the relationship between the variables and the output using a RS methodology (based on the experimental data).
- 3. Employ an efficient optimization search technique for determining the best parameter values that minimize/maximize the output value.

# 3.3.2 The Taguchi Method

The use of statistical methods in engineering can be attributed to two mathematicians in the earlier part of the 20th century (1920s): Sir R. A. Fisher in the U.K. (who first developed the DOE technique) and W. A. Shewhart in the U.S.A. (who developed the process control charts used today in statistical process control—SPC). G. Taguchi's contribution to the field can be traced to the early 1950s during his employment period by Nippon Telephone and Telegraph. During this early period, Taguchi advocated the use of orthogonal arrays in order to reduce significantly the number of experiments dictated by a full-factorial experimental design. For example, in a design problem of 13 parameters each to be evaluated at 3 different levels, we would have to run 1,594,323 experiments, whereas Taguchi's orthogonal arrays would require only 27 trials.

In the above context, the Taguchi method for parameter design aims at choosing the levels of the control variables that are robust to environmental noise. As a complementary approach to this selection of parameter values, Taguchi also proposed a technique for choosing corresponding tolerance values aimed at maximizing the quality of the manufactured product.

#### Robust Parameter Design

In Taguchi's approach, design parameters of a product are referred to as "controllable" factors versus noise factors that refer to disturbances, which cannot be controlled. The objective is, thus, to select optimal design parameter values that are least affected by noise to be encountered during the future utilization of the product. The method is a simple approach to selecting parameter values that maximize a signal-to-noise ratio (a term borrowed from the communications engineering field) defined as

$$S/N = 10 \log \left(\frac{\mu^2}{\sigma^2}\right) \tag{3.6}$$

where the mean,  $(\mu)$ , and variance,  $(\sigma^2)$ , values of the output, y (for a set of constant parameter values and n different noise levels) are defined as

$$\mu_i = \frac{1}{n} \sum_{j=1}^n y_{ij}$$
 and  $\sigma_i^2 = \frac{1}{n} \sum_{j=1}^n (y_{ij} - \mu_i)^2$  (3.7)

Based on the above definitions, Taguchi's S/N ratio encapsulates both the mean value and the variance of the output values in a single term that can be optimized by varying the design parameter values (controllable factors):

$$(S/N)_i = -10 \log \left(\frac{1}{n} \sum_{j=1}^n y_{ij}^2\right)$$
 (3.8)

That is, instead of maximizing/minimizing the mean value of the output only, Taguchi's S/N value can be used simultaneously to minimize the effect of noise (variance) on this mean value. In a DOE process based on Taguchi's method, orthogonal arrays are utilized to vary both the parameter values and the noise factors.

Let us consider a new example, in which a product's desired output is affected by four parameters, which may in turn be influenced by three noise sources. The experimental design for this example is shown in Table 2.

	Parameter set, i			et, <i>i</i>	Output @ noise set, j				(S/N) <sub>i</sub> ratio
Experiment # (i)	A	В	С	D	(1,1,1)	(1,2,2)	(2,2,1)	(2,1,2)	Eq. (3.8)
1	1	1	1	1	<b>y</b> 11	<b>y</b> <sub>12</sub>	<b>y</b> <sub>13</sub>	<b>y</b> <sub>14</sub>	( <i>S</i> / <i>N</i> ) <sub>1</sub>
2	1	2	2	2	<b>y</b> <sub>21</sub>	<b>y</b> <sub>22</sub>	<b>y</b> <sub>23</sub>	<b>y</b> <sub>24</sub>	( <b>S</b> / <b>N</b> ) <sub>2</sub>
3	1	3	3	3	<b>y</b> <sub>31</sub>	<b>y</b> <sub>32</sub>	<b>y</b> <sub>33</sub>	<b>y</b> <sub>34</sub>	( <b>S</b> / <b>N</b> ) <sub>3</sub>
4	2	1	2	3	<b>y</b> <sub>41</sub>	<b>y</b> <sub>42</sub>	<b>y</b> <sub>43</sub>	<b>y</b> 44	( <i>S</i> / <i>N</i> ) <sub>4</sub>
5	2	2	3	1	<b>y</b> <sub>51</sub>	<b>y</b> <sub>52</sub>	<b>y</b> 53	<b>y</b> 54	( <b>S</b> / <b>N</b> ) <sub>5</sub>
6	2	3	1	2	<b>y</b> <sub>61</sub>	<b>y</b> <sub>62</sub>	<b>y</b> <sub>63</sub>	<b>y</b> <sub>64</sub>	( <b>S</b> / <b>N</b> ) <sub>6</sub>
7	3	1	3	2	<b>y</b> <sub>71</sub>	<b>y</b> <sub>72</sub>	<b>y</b> <sub>73</sub>	<b>y</b> <sub>74</sub>	$(S/N)_{7}$
8	3	2	1	3	<b>y</b> <sub>81</sub>	<b>y</b> <sub>82</sub>	<b>y</b> <sub>83</sub>	<b>y</b> <sub>84</sub>	( <i>S</i> / <i>N</i> ) <sub>8</sub>
9	3	3	2	1	<b>y</b> <sub>91</sub>	<b>y</b> <sub>92</sub>	<b>y</b> 93	<b>y</b> 94	( <i>S</i> / <i>N</i> ) <sub>9</sub>

**TABLE 2**L9 and L4 Orthogonal Arrays

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In Table 2, one can notice that, the nine experiments  $(L_9)$  selected based on the three levels of four parameter values must each be repeated for two levels of three noise values  $(L_4)$  yielding a total of 36 experiments, as opposed to a total of  $3^4 \times 2^3 = 648$  (full-factorial) combinations. Once the nine S/Nvalues have been determined, a response surface is fitted to these nine data points—(A, B, C, and D)<sub>i</sub> versus the  $(S/N)_i$  values. A search through the fivedimensional response surface, then, determines the optimal parameter values for (A, B, C, and D), where the function value to be maximized is the S/N ratio.

#### **Tolerance Design**

Taguchi defines quality as an inverse function of a desired characteristic of a product and treats it as a loss. That is, every product has an associated quality loss, which could be zero if the product has the exact (expected) characteristic from the consumer's point of view. This quality "loss function" (for the output) is defined by L(y). If one assumes that process noise is normally distributed and that the mean value of this distribution is the expected output value by the customer—nominal is the best—the loss function can be defined as

$$L(y) = k(y - \mu)^{2}$$
(3.9)

where k is a cost coefficient defined by

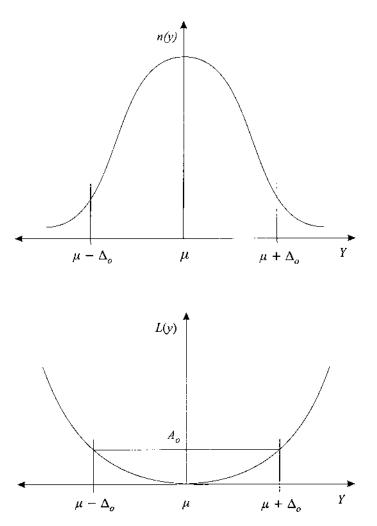
$$k = \frac{\text{consumer's loss}}{\text{functional tolerance}} = \frac{A_0}{\Delta_0^2}$$
(3.10)

The above concept of loss experienced by a customer, who expects the product to yield an output equal in magnitude to the mean, but which is actually a distance away from the mean  $(y - \mu)$ , is shown in Fig. 9. The product's output value, y, although it could be within its functional tolerance limits, defined by  $\mu \pm \Delta_0$ , still represents a loss to the customer, since it is not exactly equal to the mean value, where L(y = m) = 0. Naturally, the coefficient k in Eq. (3.9) differs from product to product and would include cost elements such as replacement, repair, service, customer loyalty, etc. Thus it is difficult to measure.

Taguchi proposes further tightening of the functional tolerance interval, specified for the desired output of the product according to the cost coefficient, k, by the cost of "fixing" the "problem" before it is shipped to the customer, as

$$\Delta = \left(\frac{A}{A_0}\right)^{1/2} \Delta_0 \tag{3.11}$$

where  $\Delta$  is the new tolerance tightened according to the cost of fixing the deviation in-house, A. If one would assume that fixing a problem before the



**FIGURE 9** Quality loss function, L(y), for a normally distributed, n(y), output.

product is shipped would cost the manufacturer less than the consequence (cost of lost quality to the customer), i.e.,  $A < A_0$ , then  $\Delta < \Delta_0$ . Thus, manufacturers should choose their process variability (variance of n(y) Fig. 9) according to the tightened tolerance limits,  $\Delta$ , and not according to  $\Delta_0$ .

In parameter design, once the tightened tolerance level,  $\Delta$ , is determined for the product's output, y, it must be propagated downward for specifying tolerances on individual (optimal) parameter values, so that their combination yields the expected  $\Delta$  on the product's output.

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The overall conclusion of Taguchi's studies has been that manufacturers must minimize the variability of their process as much as is economically viable, since customers do experience a loss in quality when they do not receive the mean output value that they expect. The tighter the process variability is, the higher the percentage of products within the (engineering) tolerance limits would be, and furthermore, the less the total cost of quality loss, which can be defined as the integral of L(y)from  $y = \mu - \Delta$  to  $\mu + \Delta$ . The product quality topic is further discussed in Chapter 16 of this book.

#### 3.4 GROUP-TECHNOLOGY-BASED DESIGN

Group technology (GT) was first proposed and developed in Europe (prior to WWII) and exported to North America in the later decades of the 20th century with the start of widespread implementation of flexible manufacturing systems (FMSs). Although it has primarily been proposed for the increased efficiency of manufacturing activities, GT can be very effectively applied to engineering design. The premise of GT philosophy is fast access to pertinent (similar) historical data available within the enterprise and its modification for the design and fabrication of new parts within the same family. In this chapter, we will concentrate on the benefits of GT in the design of products.

### 3.4.1 History of Group Technology

It has been commonly agreed upon that S. P. Mitrofanov (of the former USSR) is the originator of GT. It is also accepted that this development was based on the earlier work of A. P. Sokolovski (also of the former USSR) in the 1930s, who argued that "parts of similar geometry and materials should be manufactured in the same way by standardized technological processes." Mitrofanov elaborated on this definition by advocating the use of physical cells (machines placed in closed proximity).

The initial work of Mitrofanov was adopted in the U.K. by E. G. Brisch in the 1950s, who later with Birn developed the Brisch and Birn coding and classification method. Next, in the 1960s, came the work of Prof. H. Opitz (of the former Federal Republic of Germany), who developed the most commonly used GT system (OPITZ) in Europe. Opitz's work was originally targeted for the investigation of part statistics in the machine-tool industry. However, since then it has been used for design retrieval, process planning, and cell formation. Other GT developments in Europe included the VUOSO system developed in the former Czechoslovakia for the optimization of machine tool design, the PGM system developed in Sweden

for design retrieval, and The IAMA system developed in the former Yugoslavia for manufacturing (very similar to OPITZ). All these efforts occurred in the 1960s.

The widespread utilization of GT in the U.S.A. started first with the adaptation of the BRISCH–BIRN system named CODE for specifying geometry and function. The commercialization of the MICLASS (Metal Institute classification system) GT system, originally developed in the Netherlands by the Organization for Applied Scientific Research (TNO) in the 1970s, followed.

#### 3.4.2 Classification

Classification is the most important element of GT—it refers to a logical and systematic way of grouping things based on their similarities but then subgrouping them according to their differences. The four principles developed by Brisch for the classification of a population of parts are as follows:

- *All-embracing*: The adopted classification system must be inclusive. It must classify all current parts within the population at hand and also allow for future product features.
- *Mutually exclusive*: Once the classification structure has been developed, a part should have only one class to be included within. The system must be mutually exclusive for achieving an unambiguous distribution of parts.
- *Based on permanent features*: The classification system must utilize only the final geometrical features of the part and not any intermediate shapes.
- *From a user's point of view*: The rules of classification must be obvious to the users, and thus should be developed based on extensive interviews with all designers within the company.

The first step in implementing a classification system is a detailed review of past products and identification of primary similarities according to, for example, overall geometry (rotational versus prismatic), presence of external features (grooves, key slots, etc.) or internal features (holes, threads, etc.). Uniformity of class sizes is desirable, but owing to increased speeds of current computers, which can search databases very quickly, it is no longer a necessity. Once a representative set of historical data has been examined, and the overall classes have been determined, the next step is examining each class for differences. This step is the most critical task in classification—one must look for representative features

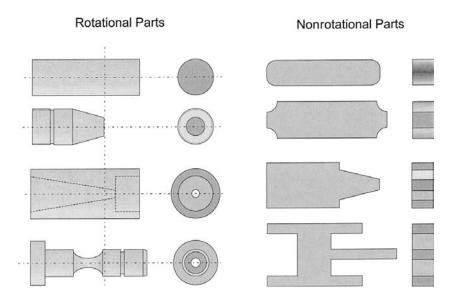


FIGURE 10 Geometrically similar parts.

that will differentiate parts and not for unique features that may never be encountered in other parts. That is, one would, actually, expect these features to be found on other past or future parts, so that when we eventually search our database we would discover past parts with similar characteristics and start our new design based on the utilization of a most similar past part—one that has the maximum number of similar features (Fig. 10).

A second level of features in a GT system would include ratios of diameter to length for rotational parts or ratios of maximum-dimension-tominimum-dimension for prismatic parts (but rarely actual dimensions). Other features could be the presence of external or internal steps, specific shapes of external or internal features, presence of threads/teeth, etc. One should recall that classification at the first or subsequent levels of features may consider characteristics, such as material type, surface finish, and tolerances, which would not be very useful to geometric modeling of a part, but critical for the use of GT in process planning and assignment of parts to certain manufacturing workcells.

In conclusion to the above classification discussion, one must note that future users of GT can easily develop their own classification system after a careful review of the literature or past developments. There exist only a very few available commercial GT systems, and these should never be treated as turnkey systems. Classification is best achieved by expert, inhouse designers.

# 3.4.3 Coding

Coding in the context of GT refers to the utilization of an alphanumeric system that will allow us to access past data with maximum efficiency. A GT coding procedure must be logical and concise. Below is a list of guidelines developed by Brisch and partners:

- A code should not exceed five characters in length without a break in the string.
- A code should be of fixed length and pattern.
- All-numeric codes are preferable—causing fewer errors.
- Alphanumeric (mixed) codes must have fixed fields for alphabetic and numeric codes, respectively—though they should be avoided if possible.

As examples, consider the following postal codes: a five-digit code followed by an additional 4-digit code for the U.S. (e.g., 17123-9254) versus the two-part, three-digit alphanumeric code in Canada (e.g., M5S 3G8).

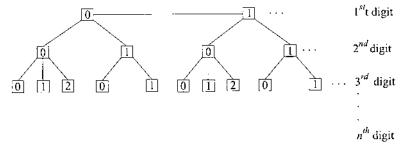
All GT-based systems would have one of the following coding structures (Fig. 11):

- *Monocode*: It can store a large amount of information within a short (length) code due to its hierarchical structure. That is, the meaning of any digit in the code is dependent on the value of the preceding digit, resulting in a tree-structure representation of a product's characteristics. Monocodes cannot be easily interpreted by people by simply examining the long code of a part. However, such codes could be easily decoded by computers.
- *Polycode:* This can only store a limited amount of information, since each character is of fixed meaning—i.e., reserved (fixed) attribute. Although easily recognizable by people (in meaning), such codes can be excessive in length.
- *Hybrid*: This is a mixture of mono- and polycodes. That is, it has a mixed structure, in which some fields have reserved (fixed) attribute meanings, regardless of the meaning of the preceding digits. Hybrid codes can be utilized for classification systems that yield group sizes of nonuniform size.

Let us now briefly review the (hybrid) MICLASS system as an example: it comprises two primary sections, a 12-digit first part and an

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# Monocode



Polycode

Hybrid Code

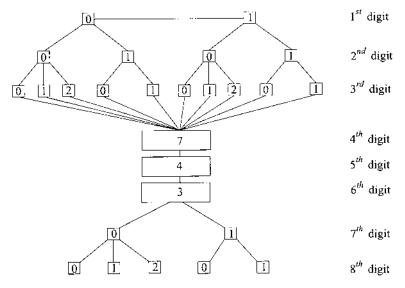


FIGURE 11 Coding systems.

additional 18-digit second part. The first four digits represent the overall geometry of a part:

1 <sup>st</sup> digit 2 <sup>nd</sup> and 3 <sup>rd</sup> digits 4 <sup>th</sup> digit	Main shape (rotational, nonrotational, special) Shape elements (stepped, with threads)
$5^{\text{th}}$ and $6^{\text{th}}$ digits	Position of shape elements Main dimensions (maximum dimension less
5 and 6 digits	than X)
7 <sup>th</sup> digit	Dimension ratio
8 <sup>th</sup> digit	Auxiliary dimension
9 <sup>th</sup> and 10 <sup>th</sup> digits	Tolerance codes (range of tolerance on dimen- sions and surface finish)
11 <sup>th</sup> and 12 <sup>th</sup> digits	Material codes (nonferrous versus composite)

# 3.4.4 Implementation

Anyone who has visited design departments of small-batch manufacturing companies, up to the early 1980s, would have noticed large numbers of cabinets full of part drawings stored for potential future use, but later completely forgotten about (or "lost in the pile") due to the lack of a logical classification and coding system. With the introduction of CAD systems, after the 1970s, the filing cabinets have been complemented, first with large numbers of magnetic tapes, later with (soft) magnetic discs, and finally with today's common hard drives. When proposing GT systems to these companies, in early 1970s, a common reply received was that they indeed did have a coding system that assigned numbers and names to these product designs, which were recorded on a log book! GT, however, is based on the utilization of a classification and coding system that would allow designers to access earlier product designs based on a select set of similarities and not simply sequentially numbering these or naming them according to their functions. Thus, today, one must still emphasize this principle in attempting to sell GT to manufacturing engineers.

Once a company has developed and installed a GT system, the first decision at hand is how to start. If it is economically feasible, a large set of past products should be coded (this step can take 1 to 6 person-months, depending on the availability of a menu-driven computer-based coding system as well as on the amount of information to be stored). An alternative would be simply to code only new parts—which would postpone a mean-ingful usage of the GT system by at least one year.

With the availability of an effective database of past designs, a designer can code a new part, based on available sketchy information, and request the GT system to identify and retrieve the most similar part

model from the database. The designer must subsequently decide whether it would be more economical to modify this past model rather than starting from scratch. The worst case scenario is the time wasted on the search that would, normally, take less than a few minutes. The time spent on coding the new (future) part is not wasted, since this code will be used when storing the new part in the database.

Several other points can be made at this time of discussing the use of GT in design efforts.

- Classification of a population of similar products can help manufacturers in standardizing parts or even deciding on how to modularize their products.
- If the GT system does include a component for process planning, fixture selection, and other manufacturing issues, at the time of information retrieval for the most similar past product, the product development team can concurrently review these pieces of information as well and make more educated design decisions on manufacturability, etc.
- GT classification and coding systems could be used in conjunction with other methods developed for feature-based design, where CAD-based solid models are automatically analyzed for similar geometric (form) features, as will be discussed in Chapter 4.

### **REVIEW QUESTIONS**

- 1. In the context of axiomatic design, define the functional requirements (FRs) and design parameters (DPs) of a product. As examples, consider two household/office products and define their FRs and DPs.
- 2. Review the two design axioms proposed by Suh and attempt to propose a third axiom that would be independent of and add value to the original two.
- **3.** In the context of axiomatic design, describe the following three design classifications: uncoupled design, coupled design, and decoupled design. Which would you prefer and why?
- 4. What is the primary purpose of a design-for-manufacturing approach? Describe one design-for-manufacturing guideline each for casting, injection molding, forging, and machining.
- 5. What is the primary purpose of a design-for-assembly approach? As examples, consider two multicomponent household/office products and comment on whether they were designed for assembly efficiency.

- 6. What is the primary purpose of a design-for-environment approach? As an example consider the use of disposable coffee/tea cups versus the use of washable (long-term, reusable) mugs. Some argue that the latter consume more resources to manufacture and maintain than the former. Discuss both sides of the argument.
- 7. Multicomponent products should be designed for recycling. Describe the three most common forms of recycling.
- **8.** Describe the data-mining activity in analyzing failed products and its benefit as a feedback tool for the design of future products.
- **9.** What is parameter and tolerance design? What is design of experiments (DOE)? What is response-surface optimization? How can product design benefit from DOE?
- **10.** Describe the Taguchi method/approach to DOE and review his proposed parameter-design and tolerance-design activities.
- **11.** What is the primary purpose of a group-technology (GT) based design approach?
- 12. Describe the typical steps of implementing a GT-based design strategy in a job-shop type manufacturing enterprise that designs and fabricates similar-geometry, make-to-order products, for example injection molds for thin-walled plastic containers for the foodpackaging industry.

# **DISCUSSION QUESTIONS**

- 1. Axiomatic design rightfully argues that products/systems must be designed based on two fundamental axioms for efficient manufacturing and effective utilization by the customer. The functional requirements of the product/system should be addressed as independently as possible, and simplicity of design should be an important objective. Review several products/systems that have been, or could have been, designed and manufactured in accordance with these axioms (e.g., light dimmers that are also used as on/off switches, auto-focus cameras, etc.). Discuss whether these axioms might not always necessarily lead to better (profitable) designs for all products/systems. Provide an example if you agree with this statement.
- 2. Quality improvement is a manufacturing strategy that should be adopted by all enterprises. Although quality control is a primary concern for any manufacturing company, engineers should attempt to improve quality. In statistical terms, all variances should be minimized, and furthermore, where applicable, the mean values should be increased (e.g., product life, strength, etc.) or decreased (e.g.,

weight) appropriately. Discuss the quality-improvement issue and suggest ways of achieving continual improvements. Discuss also whether companies should concentrate on gaining market share through improved product performance or/and quality or only through cost/price.

- **3.** Composite materials have been increasingly developed and used widely owing to their improved mechanical/electrical/chemical properties when compared to their base (matrix) material. For example, the use of glass, carbon, and Kevlar fibers in polymer base composites has significantly increased their employment in the automotive products and sports products industries. The concept of composite materials, however, may be in direct conflict with environmental and other concerns, which advocate that products should be designed so that material mix is minimized or totally avoided for ease of manufacturing and/or recycling (including decomposition) purposes. Discuss the above issues in favor of continuing to use composite materials; otherwise, propose alternatives.
- 4. Design of experiments (DOE) is a statistical approach that can be used in the design of physical or simulation-based experiments for the determination of optimal variable values. Such factorial-based experiments help engineers in the narrowing of the field of search to those parameters that have the greatest impact on the performance of the product as well as limiting the combinatoric number of variations of these variables. Discuss the role of DOE in the overall design (synthesis/ analysis) of a product, while stating advantages, benefits, etc.
- **5.** Discuss the need of developing a GT-based classification and coding system in-house as opposed to purchasing an already available (generic) commercial package.
- 6. Would several different GT-based classification and coding systems be needed in a company for different objectives? That is, would one system be needed for design, one system for manufacturing planning and yet another for cost engineering?
- 7. Group technology (GT) relies on the availability of an efficient coding system that could identify past similar product codes. Naturally, the corresponding search engine must be designed so that it closely follows the classification method employed. Two codes that are identical except for a single digit that differs by one value (e.g., 24708 versus 23708) could refer to totally dissimilar parts. Discuss the process of (tentatively) GT coding a new part under development (not in existence yet), searching the company database for the most similar past product, and proceeding from that point. Recall that two parts having identical GT codes are just "most similar" and not necessarily identical.

- 8. Failed products may provide very valuable information to manufacturers for immediate corrective actions on the design and manufacturing of current and/or future lines of products. Discuss how would you collect and analyze product failure (or survival) data for industries such as passenger vehicles, children's toys, and computer software.
- **9.** Discuss the concept of progressively increasing cost of changes to a product as it moves from the design stage to full production and distribution. How could you minimize necessary design changes to a product, especially for those that have very short development cycles, such as portable communication devices?

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